Nonhomogeneous magnetization and superconductivity in superconductor-ferromagnet structures

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Abstract

We study two different superconductor-ferromagnet (S/F) structures. We consider first a Josephson junction which consists of two S/F bilayers separated by an insulating layer. We show that for an antiparallel alignment of the magnetization in the two F layers the Josephson critical current I_c increases with increasing exchange field h. The second system we consider is a S/F structure with a local inhomogeneity of the magnetization in the ferromagnet near the S/F interface. Due to the proximity effect not only a singlet but also a triplet component of the superconducting condensate is induced in the ferromagnet. The latter penetrates over the length $\sqrt{D/\epsilon}$ (D is the diffusion coefficient and ϵ the energy). In the case of temperatures of the order of the Thouless energy this length is comparable to the length of the ferromagnet. This long-range penetration leads to a significant increase of the ferromagnet conductance below the superconducting critical temperature T_c. Contrary to the case of the singlet component, the contribution to the conductance due to the odd triplet component is not zero at $T = 0$ and $V = 0$ (V is the voltage) and decays with increasing temperature T in a monotonic way.

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1 Introduction

The interplay between superconductivity and ferromagnetism has been the subject of extensive research for many years. The ferromagnetism, being usually much stronger than superconductivity, is supposed to destroy the latter. This suppression is caused by two mechanisms. One of them is related to the internal magnetic field which is created by the ordered magnetic moments. The internal magnetic field, which is proportional to the magnetization M, induces Meissner currents and suppresses the superconducting order parameter Δ. This mechanism was first analyzed by Ginsburg [1]. He concluded that the coexistence phase could not take place in an ordinary bulk sample. The second mechanism is due to the direct action of an exchange field h on spins of electrons[2]. The exchange field tends to align the spins of the electrons in one direction and thereby destroys the singlet Cooper pairs. The influence of the magnetization on the orbital condensate motion may be reduced drastically if the magnetization changes its direction on the scale of the superconducting correlation length in such a way that the averaged magnetic field is zero (as in the case of a spiral magnetic structure). The action of the exchange field may be also decreased if it is realized via the RKKY interaction [3] (for details see the review [5]). The coexistence of superconductivity and ferromagnetism was observed in ternary rare earth compounds [4].

The layered superconductor/ferromagnet (S/F) structures open new possibilities to achieve the coexistence of ferromagnetism and superconductivity. In such systems ferromagnetic and superconducting regions are spatially separated; therefore if the magnetization is oriented parallel to the layers, it does not strongly affect the condensate in the S layers. On the other hand the Cooper pairs penetrates into the F layer over a length which in the dirty limit ($h\tau \ll 1$, τ is the elastic scattering time) is of the order of $\sqrt{D/h}$, where D is the diffusion coefficient in the F layer. The S/F structure becomes superconducting if the F layers are thin enough. Layered structures can be created artificially or can be found in some high-T_c cuprates, as in the compound $RSr_2RCu_3O_8$ (R: rare earth). Its structure is similar to the one of the high-$T_c$material $YBa_2Cu_3O_7$. Magnetic ordering in RuO_2 layers (it is assumed that mostly an antiferromagnetic order is realized) occurs at $T_M = 133K$ and superconducting transition presumably in CuO_2 layers occurs at $T_c \simeq 50K$ [6]. It is interesting to note that another material from the rutinate class, Sr_2RuO_4, is an exotic triplet superconductor [7]. A few interesting effects were predicted and observed in layered S/F structures. Since the experimental pioneering work performed by Werthamer et al. [8], it is known that the superconducting critical temperature T_c is lowered in S/F multilayered structure with increasing thickness of the F layers. This is due to the proximity effect, i.e. superconductivity in the S layer is suppressed to some extent if it is brought into contact with a non-superconducting (specially magnetic) layer. Another interesting phenomenon which can occur in S/F layered structures is an oscillatory dependence of the critical temperature T_c and

the critical current on the thickness of the F layer [9, 10, 11, 12]. These oscillations are related to oscillations of the condensate in the F layers which can be formally obtained by the replacement $\omega \to \omega + ih$ (here ω is the Matsubara frequency which enters in the expression for the condensate Green's functions). An interesting behavior of the critical Josephson current I_c as a function of the exchange field h was predicted first by Bulaevskii et al. [9]; they found that with increasing h the current I_c changes sign and the phase difference $\varphi = \pi$ is established in a Josephson S/F/S junction (the so called π-junction). This prediction was confirmed in a recent experiment [13].

Despite of a large number of theoretical works on equilibrium and nonequilibrium effects in S/F/S structures, some important features of the effects remain unclear. In this report we discuss the influence of different relative orientations of the magnetizations on the critical Josephson current in a tunnel SF/I/FS junction. Its electrodes consist of SF bilayers. The critical current I_c is not zero if the thickness of the ferromagnetic F layer is thin enough and depends on relative orientation and absolute values of magnetization or exchange energy h. Surprisingly the current I_c turns out to be not decreasing, but increasing function of h in the case of antiferromagnetic configuration (the magnetization vectors are aligned in opposite directions in the electrodes). Therefore the critical current at nonzero h exceeds its value in the absence of h.

The second problem which we discuss here is a possible mechanism of the enhanced conductance measured experimentally in mesoscopic S/F structures. Recent experiments on S/F structures showed that below the critical temperature T_c the conductance of the ferromagnetic wire (or film) varies with the temperature in a nonmonotonic way and may exceed its value above T_c [14, 15, 16]. The decrease of the conductance, which was also observed, has been explained in a few theoretical papers [17, 18, 19]. Although it was assumed in some papers [17, 19] that an increase in the conductance may be due to scattering at the S/F interface, careful measurements demonstrated that the entire change of the conductance is due to an increase of the conductivity of the ferromagnet [14, 15]. Such an increase would not be a great surprise if instead of the ferromagnet one had a normal metal N. It is well known (see for review [17, 20]) that in S/N structures the proximity effect can lead to a considerable increase of the conductance of the N wire provided its length does not exceed the phase breaking length L_φ. However, in a S/F structure if the superconducting pairing is singlet, the proximity effect is negligible at distances exceeding a much shorter length $\xi_h \sim \sqrt{D/h}$, where h is the exchange energy in the ferromagnet. This reduction of the proximity effect due to the exchange field of the ferromagnet is clear from the picture of Cooper pairs consisting of electrons with opposite spins. The proximity effect is not considerably affected by the exchange energy only if the latter is small enough, *i.e.* $h < T_c$. In the experiments of Refs.[14, 15] strong ferromagnets, as Ni or Co, were used. Their exchange energy h is by several orders of magnitude larger than T_c, and therefore a singlet pairing is impossible due to the strong difference in the energy dispersions of the two spin

bands. At the same time, an arbitrary exchange field cannot destroy a triplet superconducting pairing because the spins of the electrons forming Cooper pairs are already parallel. We suggest a new mechanism for the increase of the conductance in S/F structures. This mechanism is based on the formation of the triplet component in the F wire, which is due to a local inhomogeneity of the magnetization M in the vicinity of the S/F interface. We show that the inhomogeneity generates a triplet component of the superconducting order parameter with an amplitude comparable with that of the singlet pairing. The penetration length of the triplet component into the ferromagnet is equal to $\xi_\varepsilon = \sqrt{D/\varepsilon}$, where the energy ε is of the order of temperature T or the Thouless energy $E_T = D/L^2$, L is the sample size. The length ξ_ε is of the same order as that for the penetration of the superconducting pairs into a normal metal and therefore the increase of the conductance due to the proximity effect can be compared to that in an S/N structure. The inhomogeneity may appear in a natural way as an edge effect or may be created intentionally. The triplet component we considered differs from that in Sr_2RuO_4; it is almost symmetrical in momentum p (the antisymmetrical part is small) and is an odd function of the Matsubara frequency ω. This type of the odd triplet component was for the first time suggested by Berezinskii [21] as a possible condensate phase in superfluid He^3 (in fact a condensate function symmetrical in ω and antisymmetrical in momentum $\overrightarrow{p}$ takes place in He^3). Later the so called odd superconductivity was discussed in Refs.[22] as a possible mechanism for high T_c superconductivity. This odd in ω and even in $\overrightarrow{p}$ condensate component is not suppressed by the impurity scattering.

2 The SF/I/FS junction

In this section we consider a layered system consisting of two F/S bilayers separated by an insulating layer (see Fig.1). In this case the Josephson critical current is determined by the transparency of the insulating layer and depends on the relative orientation of magnetization in the F layers. We assume that the F and the S layers $d_{F,S}$ are thin enough: $d_{F,S} < \xi_{F,S}$, where $\xi_F = \sqrt{D/h}$ and $\xi_S = \sqrt{D/\Delta}$. With this assumption one comes to effective values of the superconducting order parameter Δ_{eff}, of the coupling constant λ_{eff}, and of the magnetic moment h_F described by the following equations

$$\Delta_{eff}/\Delta = \lambda_{eff}/\lambda = \nu_s d_s \left(\nu_s d_s + \nu_f d_f\right)^{-1},$$
$$h_F/h = \nu_f d_f \left(\nu_s d_s + \nu_f d_f\right)^{-1} \tag{1}$$

where ν_s and ν_f are the densities of states in the superconductor and ferromagnet, respectively.

First, we analyze the case of a high S/F interface transparency, i.e. $R_{S/F} < \rho_F/\xi_F$. Under these conditions all the Green's functions are nearly constant in space and continuous across the S/F interface.

In order to find the Green's functions we use the Usadel equation [23], which in the presence of a nonhomogeneous exchange field has the general form

$$-iD\nabla\left(\breve{\mathbf{g}}\nabla\breve{\mathbf{g}}\right)+i\left(\hat{\tau}_3\otimes\hat{\sigma}_0.\partial_t\breve{\mathbf{g}}+\partial_{t'}\breve{g}.\hat{\tau}_3\otimes\hat{\sigma}_0\right)+eV(t)\breve{\mathbf{g}}-\breve{\mathbf{g}}eV(t')+\left[\hat{\Delta}\otimes\hat{\sigma}_3,\breve{\mathbf{g}}\right]+\left[\check{M}_h,\breve{\mathbf{g}}\right]=0 \tag{2}$$

Here $\breve{\mathbf{g}}$ is the quasiclassical Green's function, which has the form

$$\breve{\mathbf{g}}=\begin{pmatrix}\breve{g}^R & \breve{g}^K\\ 0 & \breve{g}^A\end{pmatrix}, \tag{3}$$

$\check{M}_h=h\left(\hat{\tau}_3\otimes\hat{\sigma}_3\cos\alpha+\hat{\tau}_0\otimes\hat{\sigma}_2\sin\alpha\right)$,

$$\hat{\Delta}=\begin{pmatrix}0 & \Delta\\ -\Delta^* & 0\end{pmatrix},$$

and the matrices $\hat{\tau}_i$ and $\hat{\sigma}_i$ are the Pauli matrices in the Nambu and spin space respectively; $i=0,1,2,3$, where $\hat{\tau}_0$ and σ_0 are the corresponding unit matrices. The Eq.(2) is supplemented by the normalization condition

$$\breve{\mathbf{g}}.\breve{\mathbf{g}}=\check{1}\ . \tag{4}$$

The current density is determined by the usual expression

$$I_J=\frac{1}{16\rho}\mathrm{Tr}\left(\hat{\tau}_3\otimes\hat{\sigma}_0\right)\int \mathrm{d}\epsilon\left(\breve{g}^R.\partial_x\breve{g}^K+\breve{g}^K.\partial_x\breve{g}^A\right)\ . \tag{5}$$

In order to find the Green's functions $\breve{g}^{R(A)}$, we multiply the components (1,1) and (2,2) of the matrix equation (2) (the Usadel equations) by the density of states $\nu_{F,S}$ in the F and the S layers respectively, and integrate over the thickness of the bilayers. Neglecting the influence of one bilayer on the other (this means that $(\breve{g}\partial_x\breve{g})=0$ at the F/I interface), we obtain, in the Matsubara representation, the following equation:

$$\left[\check{M},\breve{g}\right]+\left[\hat{\Delta}_{eff}\otimes\hat{\sigma}_3,\breve{g}\right]=0\ , \tag{6}$$

Here $\breve{g}$ denotes the Matsubara Green's function, and the matrix $\check{M}$ is given by

$$\check{M}=\hat{\tau}_3\otimes\left(\hat{\sigma}_0 i|\omega_m|+\hat{\sigma}_3 h_{eff}\mathrm{sgn}\omega_m\cos\alpha\right)-\hat{\tau}_0\otimes\hat{\sigma}_2 h_{eff}\mathrm{sgn}\omega_m\sin\alpha\ , \tag{7}$$

where ω_m is the Matsubara frequency. In what follows we will skip the indices eff. We assume that the vector $\mathbf{h}$ in the left layer is oriented along the z-axis and has the components $h\left(0,\sin\alpha,\cos\alpha\right)$ in the right electrode. One can simplify Eq. (6) in the right bilayer with the help of the following unitary transformation

$$\widetilde{\breve{g}}=\check{U}^{+}.\breve{g}.\check{U}\ , \tag{8}$$

where $\check{U} = \hat{\tau}_0 \otimes \hat{\sigma}_0 \cos(\alpha/2) + i \sin(\alpha/2) \hat{\tau}_3 \otimes \hat{\sigma}_1$. In this case one obtains for the both layers the same equation:

$$[\hat{\tau}_3 \otimes (\epsilon \hat{\sigma}_0 + h_F \hat{\sigma}_3), \check{g}] + \left[\hat{\Delta}_S \otimes \hat{\sigma}_3, \check{g}\right] = 0 \ . \tag{9}$$

We can solve Eq.(9) by making the ansatz

$$\check{g} = \hat{\tau}_3 \otimes (a_0 \hat{\sigma}_0 + a_3 \hat{\sigma}_3) + \hat{\Delta}_S \otimes (b_0 \hat{\sigma}_0 + b_3 \hat{\sigma}_3) \,. \tag{10}$$

From Eq. (9) and the normalization condition (4) one can obtain the coefficients a's and b's. In the left bilayer $\check{g}$ is given by the expression (10) while in the right bilayer it is given by $\check{g}^{(r)} = \check{U}^{+} \widetilde{\check{g}}^{(r)} \check{U}$, i.e.

$$\check{g}^{(r)} = \hat{\tau}_3 \otimes (a_0 \hat{\sigma}_0 + a_3 \cos\theta \hat{\sigma}_3) - \hat{\tau}_0 \otimes a_3 \sin\theta \hat{\sigma}_2 + \hat{\Delta}_S \otimes (b_0 \cos\theta \hat{\sigma}_0 + b_3 \hat{\sigma}_3) - \hat{\tau}_3 \hat{\Delta}_S \otimes i b_0 \sin\theta \hat{\sigma}_1 \ .$$

According to Eq. (5) only the coefficients b_0 and b_3 will enter in the expression for the Josephson current, and they are given by

$$(b_3)_{l,r} = \frac{1}{2} \left(\frac{1}{\xi_+} + \frac{1}{\xi_-} \right)_{l,r} \text{ and } (b_0)_{l,r} = \frac{1}{2} \left(\frac{1}{\xi_+} - \frac{1}{\xi_-} \right)_{l,r} \ ,$$

where $\xi_\pm = \sqrt{\epsilon_\pm^2 - |\Delta_S|^2}$, and $\epsilon_\pm = i\omega_m \pm h$. By writing $\Delta_S = |\Delta_S| \exp(i\varphi)$ in the right side one obtains the following expression for the critical current

$$eV_c(\alpha) \equiv eI_c R_b = 2\pi T \Delta_l \Delta_r \sum_{m>0} \left\{ \mathrm{Re} \left(\frac{1}{\xi_m} \right)_l \mathrm{Re} \left(\frac{1}{\xi_m} \right)_r - \cos\theta \mathrm{Im} \left(\frac{1}{\xi_m} \right)_l \mathrm{Im} \left(\frac{1}{\xi_m} \right)_r \right\} \ , \tag{11}$$

where $\xi_n = \sqrt{(\omega_m + i h_F)^2 + \Delta_S^2}$ and R_b is the tunnel resistance of the I layer. We note that the system under consideration is equivalent to a Josephson junction consisting of two magnetic superconductors. The same structure was also analyzed in Ref.[24], where the critical current was calculated for different S/F interface transparencies. The authors have found the conditions under which the system undergoes a transition to the π state; however they analyzed only the case of parallel magnetization.

Here we consider two limiting cases: a) a parallel relative orientation of the magnetizations, i.e. $\alpha = 0$ and b) an antiparallel orientation: $\alpha = \pi$.

In the case $\alpha = 0$ according to Eq. (11), the critical current is given by the expression

$$eV_{c\uparrow\uparrow} \equiv eI_{c\uparrow\uparrow} R_b = 4\pi T \Delta_S^2 \sum_m \frac{\omega_m^2 + \Delta_S^2 - h_F^2}{(\omega_m^2 + \Delta_S^2 - h_F^2)^2 + 4\omega_m^2 h_F^2} \ . \tag{12}$$

In writing Eq. (12) we assumed that h_F and $|\Delta_S|$ are the same in both bilayers (symmetric structure). The dependence of the critical current on the exchange

field h_F is shown in Fig.2. At $T = 0$ the current I_c is constant up to the value $h_F = \Delta_0$ where it drops to zero; Δ_0 is the effective energy gap Δ_S at zero temperature and zero exchange field. This is a consequence of the fact that the order parameter Δ is also constant. We do not consider here a possible transition to the LOFF phase predicted by Larkin and Ovchinnikov (LO) [25] and Fulde and Ferrell (FF) [26] for the region $0.755\Delta_{S0} < h_F$. We argue that since the homogeneous superconducting state in this region is a metastable state, its realization is possible. Nevertheless our result is definitely valid for the region of small h_F, and a possible transition to the LOFF would manifest itself in a drop of the the critical current.

More interesting is the case when the relative orientation of the magnetizations is antiparallel, i.e. $\alpha = \pi$. This case was considered in Ref. [27]. Then, the critical current is given by the expression

$$eV_{c\uparrow\downarrow}(\pi) \equiv eI_{c\uparrow\downarrow}R_b = 4\pi T\Delta_S^2 \sum_m \frac{1}{\sqrt{\left(\omega_m^2 + \Delta_S^2 - h_F^2\right)^2 + 4\omega_m^2 h_F^2}} \,. \tag{13}$$

In this case the dependence of I_c on h_F is completely different from that given by Eq. (12) (see Fig.3). The critical current determined by Eq. (13) increases with increasing h_F (i.e. with increasing either h or d_F) and even diverges at zero temperature when $h_F \to \Delta_S$. Of course, there is no real divergence of I_c since, for example, finite temperatures smear out this divergency. The critical current has a maximum at some value of h_F close to Δ_0. With decreasing T the maximum value of I_c increases and its position is shifted towards Δ_0. For arbitrary relative orientations of magnetizations the expression for $V_c(\alpha)$ can be presented in the form

$$V_c(\alpha) = V_{c\uparrow\uparrow}\cos^2(\alpha/2) + V_{c\uparrow\downarrow}\sin^2(\alpha/2) \,. \tag{14}$$

Therefore, the singular part is always present and its contribution reaches 100% at $\alpha = \pi$.

All the conclusions given above remain valid also for two magnetic superconductors with uniformly oriented magnetizations in each layer. As in the previous case of parallel orientations, the state with $h_F = \Delta_0$ might be unreachable for the antiparallel orientation due to the appearance of the inhomogeneous LOFF state. However the singular behavior of I_c can be realized at smaller values of h in a structure with large enough S/F interface resistance $R_{S/F}$. In this case the bulk properties of the S film are not changed by the proximity of the F film (to be more precise the condition $R_{S/F} > (\nu_F d_F/\nu_S d_S)\rho_F\xi_F$ must be satisfied; ρ_F is the specific resistance of the F film). Then, as one can readily show [28], a subgap $\epsilon_{sg} = (D\rho)_F \,/\, \left(R_{S/F} d_F\right)$ arises in the F layer. The Green's functions in the F layer have the same form as in Eq. (10) with Δ_S replaced by ϵ_{sg}. The singularity in $I_c(h_F)$ occurs at h_F equals to ϵ_{sg}, and the LOFF state does not arise because the subgap ϵ_{sg} is not determined by the self-consistency equation.

Another model of a SF/I/FS Josephson junction was considered in a recent work [29]. The authors assumed that $d_s > \xi_s$ and that the conductance of the S layer in the normal state is much larger than the conductance of the F layer. According to this model the critical current may either increase or decrease as a function of h, and for certain values of h may become negative (π-junction). It is worthwhile noting that real structures are much more complicated than the models presented above. In order to obtain a complete description of the systems one should take into account that electrons with up and down spins have different density-of-states, conductivities and transmission coefficient through the S/F interface.

3 Triplet pairing and long-range proximity effect

Although in almost all known superconductors Cooper pairs are in spin singlet state, the triplet superconductivity have been studied in some works. Many years ago Berezinskii studied a possible triplet phase in superfluid ^3He [21]. The triplet component of the condensate function proposed in Ref.[21] was odd in frequency and even in momentum. However nowadays it is known that this hypothetical condensate function does not take place in ^3He. The discovery of superconductivity in Sr_2RuO_4 [7] has arisen the general interest in triplet superconductivity. In the case of S/F structures, the possible role of a triplet component in transport properties was studied in Refs. [18, 30]. In both cases the triplet component arose as a result of mesoscopic fluctuations, and the correction to the conductance were very small. In this section we suggest another mechanism of formation of triplet pairing in S/F structures. This is due to a local inhomogeneity of the magnetization in the vicinity of the S/F interface.

We consider the system shown in Fig.4 and assume that the magnetization orientation varies linearly from $\alpha = 0$ at $x = 0$ to $\alpha_w = Qw$ at $x = w$. Here α is the angle between M and the z-axis (the x-axis is parallel to the f wire). Thus in this region the magnetization is given by

$$\mathbf{M} = h\,(0, \sin Qx, \cos Qx)\ . \tag{15}$$

We also consider the diffusive limit corresponding to short mean free path and to the condition $h\tau \ll 1$. Thus we may describe the system using Eq. 2. This equation contains the normal $\check{g}$ and anomalous $\check{f}$ Green's functions, which are 4×4 matrices in the Nambu$\otimes$spin space. Due to the strong mismatch of the Fermi surfaces the transmission coefficient through the S/F interface is small and therefore we can assume that the anomalous condensate function $\check{f}$ is also small. In the case of high transparency the order parameter in the superconductor might be suppressed, and therefore it is also possible to assume a weak proximity effect. Thus in both cases we can use the linearized Usadel

equation for the retarded matrix (in spin space) Green's function $\hat{f}^R$, which has the form(the index R is dropped)

$$-iD\partial^2_{\mathbf{r}}\hat{f} + 2\epsilon\hat{f} - 2\Delta\hat{\sigma}_3 + \left(\hat{f}\hat{V}^* + \hat{V}\hat{f}\right) = 0 \ . \tag{16}$$

Here the matrix $\hat{V}$ is defined as $\hat{V} = h\left(\hat{\sigma}_3 \cos\alpha + \hat{\sigma}_2 \sin\alpha\right)$, where α varies with x as shown in Fig.4. Eq. (16) is supplemented by the boundary conditions at the interface that can also be linearized [31]. Assuming that there are no spin-flip processes at the S/F interface, we have

$$\partial_x \hat{f}\Big|_{x=0} = \left(\rho/R_b\right)\hat{f}_S \ , \tag{17}$$

where ρ is the resistivity of the ferromagnet, R_b is the S/F interface resistance per unit area in the normal state, and $f_S = \hat{\sigma}_3\Delta/\sqrt{\epsilon^2 - \Delta^2}$.

The solution of Eq. (16) is trivial in the superconductor but needs some care in the ferromagnet. In the region $0 < x < w$ the solution $\hat{f}$ can be sought in the form

$$\hat{f} = \hat{U}\left(x\right)\hat{f}_n\hat{U}\left(x\right) \ , \tag{18}$$

where $\hat{U}$ is again an unitary transformation given by $\hat{U} = (x) = \hat{\sigma}_0 \cos\left(Qx/2\right) + i\hat{\sigma}_1 \sin\left(Qx/2\right)$.

Substituting Eq. (18) into Eq. (16) and assuming that the solution depends on the coordinate x only we obtain the following equation for $\hat{f}_n$

$$-iD\partial^2_{xx}\hat{f}_n + i\left(DQ^2/2\right)\left(\hat{f}_n + \hat{\sigma}_1\hat{f}_n\hat{\sigma}_1\right) + DQ\left\{\partial_x\hat{f}_n, \hat{\sigma}_1\right\} + 2\epsilon\hat{f}_n + h\left\{\hat{\sigma}_3, \hat{f}_n\right\} = 0 \ . \tag{19}$$

Here $\{...\}$ is the anticommutator. In the region $x > w$, $\hat{f}_n$ satisfies Eq. (19) with $Q = 0$.

We see from Eq. (19) that the singlet and triplet components of the anomalous function $\hat{f}_n$ inevitably coexist in the ferromagnet. They are mixed by the rotating exchange field h. In the region $x > w$ these components decouple and their amplitudes should be found by matching the solutions at $x = w$.

Eq. (19) can be solved exactly. The solution $\hat{f}_n$ can be written in the form

$$\hat{f}_n = \hat{\sigma}_0 A\left(x\right) + \hat{\sigma}_3 B\left(x\right) + i\hat{\sigma}_1 C\left(x\right) \tag{20}$$

The function $C\left(x\right)$ in Eq. (20) is the amplitude of the triplet pairing, whereas the first and the second term describe the singlet one. Substituting Eq. (20) into Eq. (19) we obtain a system of three equations for the functions A, B and C, which can be sought in the form

$$A\left(x\right) = \sum_{i=1}^{3}\left(A_i \exp\left(-\kappa_i x\right) + \bar{A}_i \exp\left(\kappa_i x\right)\right) \tag{21}$$

The functions $B(x)$ and $C(x)$ can be written in a similar way. The eigenvalues κ_i obey the algebraic equations

$$
\begin{aligned}
\left(\kappa^2 - \kappa_\epsilon^2 - Q^2\right) C - 2\left(Q\kappa\right) A &= 0 \\
\left(\kappa^2 - \kappa_\epsilon^2\right) B - \kappa_h^2 A &= 0 \\
\left(\kappa^2 - \kappa_\epsilon^2 - Q^2\right) A - \kappa_h^2 B + 2\left(Q\kappa\right) C &= 0\;,
\end{aligned}
\tag{22}
$$

where $\kappa_\epsilon^2 = -2i\epsilon/D$ and $\kappa_h^2 = -2ih/D$ (indices i were dropped). The eigenvalues κ are the values at which the determinant of Eqs. (22) turns to zero. From the first equation of Eqs. (22) we see that in the homogeneous case ($Q = 0$) the triplet component has a characteristic penetration length $\sim \kappa_\epsilon^{-1}$, but we see from Eq. (17) that its amplitude is zero. If $Q \neq 0$, the triplet component C is coupled to the singlet component (A, B) induced in the ferromagnet according to the boundary condition Eq. (17) (proximity effect). If the width w is small, the triplet component changes only a little in the region $(0, w)$ and spreads over a large distance of the order $\left|\kappa_\epsilon^{-1}\right|$ in the region $(0, L)$. In the case of a strong exchange field h, ξ_F is very short ($\xi_F \ll w, \xi_T$), the singlet component decays very fast over the length ξ_F, and its slowly varying part turns out to be small. In this case the first two eigenvalues $\kappa_{1,2} \approx (1 \pm i)/\xi_F$ can be used everywhere in the ferromagnet ($0 < x < L$), where L is the length of the ferromagnet. As concerns the third eigenvalues, we obtain $\kappa_3 = \sqrt{\kappa_\epsilon^2 + Q^2}$ in the interval $(0, w)$, and $\kappa_3 = \kappa_\epsilon$ in the interval (w, L). The amplitude B_3 of the slowly varying part of the singlet component is equal to $B_3 = 2\left(Q\kappa_3/\kappa_h^2\right) C_3 \ll C_3$.

All the amplitudes should be chosen to satisfy the boundary conditions at $x = 0$ (Eq. (17)) and zero boundary condition at $x = L$. For the triplet component we obtain (we restore the indices R(A))

$$
C^{R(A)}(x) = \mp i\left\{QB(0)\sinh\left(\kappa_\epsilon(L-x)\right)\left[\kappa_\epsilon\cosh\Theta_\epsilon\cosh\Theta_3 + \kappa_3\sinh\Theta_\epsilon\sinh\Theta_3\right]^{-1}\right\}^{R(A)}\;,
\tag{23}
$$

where $w < x < L$, $B^{R(A)}(0) = \left(\rho\xi_h/2R_b\right) f_S^{R(A)}$ is the amplitude of the singlet component at the S/F interface, $\Theta_\epsilon = \kappa_\epsilon L$, $\Theta_3 = \kappa_3 w$, and $\kappa_\epsilon^{R(A)} = \sqrt{\mp 2i\epsilon/D}$.

It is clear from Eq. (23), that the triplet component is of the same order of magnitude as the singlet one at the interface. Indeed, for the case $w \ll L$ we obtain from Eq. (23) $|C(0)| \sim B(0)/\sinh\alpha_w$, where $\alpha_w = Qw$ is the angle characterizing the rotation of the magnetization. Therefore if the angle $\alpha_w \leq 1$ and the S/F interface transparency is not too small, the singlet and triplet components are not small. They are of the same order in the vicinity of the S/F interface, but while the singlet component decays abruptly over a short distance ($\sim \xi_F$), the triplet one varies smoothly along the ferromagnet, turning to zero at the F reservoir. In Fig.5 we plot the spatial dependence of the singlet $|B(x)|$ and the triplet $|C(x)|$ components for two different Q. One can see that the singlet component decays abruptly undergoing the well known oscillations

[32] while the triplet one decays to zero slowly. This decay in the region $(0, w)$ increases with increasing Q.

Thus, we come to a remarkable conclusion: the penetration of the superconducting condensate into a ferromagnet may be similar to the penetration into a normal metal. The only difference is that, instead of the singlet component in the case of the normal metal, the triplet one penetrates into the ferromagnet. Of course, in order to induce the triplet component one needs an inhomogeneity of the exchange field at the interface.

The presence of the condensate function (triplet component) in the ferromagnet can lead to interesting long-range effects. One of them is a change of the conductance of a ferromagnetic wire in a S/F structure (see inset in Fig.4) when the temperature is lowered below T_c. This effect was observed first in S/N structures and later was successfully explained (see, e.g. reviews [17, 20]). Now we consider the S/F structure shown in the inset of Fig.1. The normalized conductance variation $\delta\tilde{G} = (G - G_n)/G_n$ is given by the expression [33]:

$$\delta\tilde{G} = -\frac{1}{32T}\mathrm{Tr}\int \mathrm{d}\epsilon F_V' \left\langle \left[\hat{f}^R(x) - \hat{f}^A(x)\right]^2 \right\rangle \ . \tag{24}$$

Here G_n is the conductance in the normal state, $< .. >$ denotes the average over the length of the ferromagnetic wire between the F reservoirs, and F_V' is given by the expression

$$F_V' = 1/2\left[\cosh^{-2}((\epsilon + eV)/2T) + \cosh^{-2}((\epsilon - eV)/2T)\right] \ . \tag{25}$$

The function $\hat{f}$ is given by the third term of Eq. (20) with $C^R = -\left(C^A\right)^*$ (we neglect the small singlet component). Substituting Eqs. (20, 23) into Eq. (24) one can determine the temperature dependence $\delta\tilde{G}(T)$. Fig.6 shows this dependence. We see that $\delta\tilde{G}$ increases with decreasing temperature and saturates at $T = 0$. This monotonic behavior of $\delta\tilde{G}$ contrasts with the so called reentrant behavior of $\delta\tilde{G}$ in S/N structures [34, 35] and is a result of broken time-reversal symmetry of the system under consideration.

Available experimental data are still controversial. It has been established in a recent experiment [16] that the conductance of the ferromagnet does not change below T_c and all changes in δG are due to changes of the S/F interface resistance R_b. However, in other experiments R_b was negligibly small [14]. The mechanism suggested in our work may explain the long-range effects observed in the experiments [14, 15]. At the same time, the result of the experiment [16] is not necessarily at odds with our findings. The inhomogeneity of the magnetic moment at the interface, which is the crucial ingredient of our theory, is not a phenomenon under control in these experiments. One can easily imagine that such inhomogeneity existed in the structures studied in Refs. [14, 15] but was absent in those of Ref. [16]. The magnetic inhomogeneity near the interface may have different origins. Anyway, a more careful study of the possibility of a rotating magnetic moment should be performed to clarify this question.

In order to explain the reentrant behavior of $\delta G(T)$ observed in Refs. [14, 15] one should take into account other mechanisms, as those analyzed in Refs. [19, 18, 36]. However, this question is beyond the scope of the present paper.

We note that at the energies ϵ of the order of Thouless energy $\epsilon \sim E_T$ the triplet component spreads over the full length L of the ferromagnetic wire (see Fig.2). This long-range effect differs completely from the proximity effect in a ferromagnet with a uniform magnetization considered recently in Ref.[37]. In the latter case the characteristic wave vector is equal to $\kappa_{1,2} = \sqrt{-2i(\epsilon \pm h)/D}$ (cf. Eqs. (22)). It was noted in Ref. [37] that if $\epsilon \to \pm h$, then $\kappa_{1,2} \to 0$ and the singlet component penetrates in the ferromagnet. If the characteristic energies $\epsilon_{ch} \sim E_T, T$ are much less than h, the penetration length $|\kappa_{1,2}|^{-1}$ is of the order ξ_F and is much shorter then ξ_T or L.

4 Conclusion

We analyzed two effects related to a nonhomogeneous magnetization in S/F structures. First we have calculated the Josephson critical current I_c in a tunnel S/F-I-S/F junction. We have shown that in contrast to a common view, in the case of an antiparallel configuration (the magnetization vectors in the S/F electrodes are antiparallel to each other) the critical current I_c increases with the increasing exchange field h. This means that in a S/F-I-F/S junction the current I_c can be even greater than that in a S-I-S junction with the same S electrodes. Secondly we have calculated the conductance of a mesoscopic F wire attached to a superconductor S. We have shown that in the presence of a local inhomogeneity near the S/F interface, both singlet and triplet components of the condensate are created in the ferromagnetic wire due to the proximity effect. The singlet component penetrates into the ferromagnet over a short length ξ_F, whereas the triplet component can spread over the full mesoscopic length of the ferromagnet. This long-range penetration of the triplet component should lead to a significant variation of the ferromagnet conductance below T_c.

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FIG.1: The S/F-I-F/S system.

FIG.2:Dependence of the normalized critical current on h for different temperatures in the case of a parallel orientation. Here $eV_c = eRI_c$, h_F is the effective exchange field, $t = T/\Delta_0$ and Δ_0 is the superconducting order parameter at $T = 0$ and $h = 0$.

FIG.3: Dependence of the normalized critical current on h for different temperatures in the case of an antiparallel orientation. Here $eV_c = eRI_c$, h_F is the effective exchange field, $t = T/\Delta_0$ and Δ_0 is the superconducting order parameter at $T = 0$ and $h = 0$.

FIG.4: Schematic view of the structure under consideration. In the inset is shown the structure, for which we calculate the conductance variation: two ferromagnetic wires connected to two ferromagnetic and two superconducting reservoirs.

FIG.5: Spatial dependence of the singlet (dashed line) and triplet (solid line) components of $\left|\hat{f}\right|$ in the F wire for different values of α_w. Here $w = L/5$, $\epsilon = E_T$ and $h/E_T = 400$. $E_T = D/L^2$ is the Thouless energy.

FIG.6: The $\delta G(T)$ dependence. Here $\gamma = \rho\xi_h/R_b$. $\Delta/E_T \gg 1$ and $w/L = 0.05$.

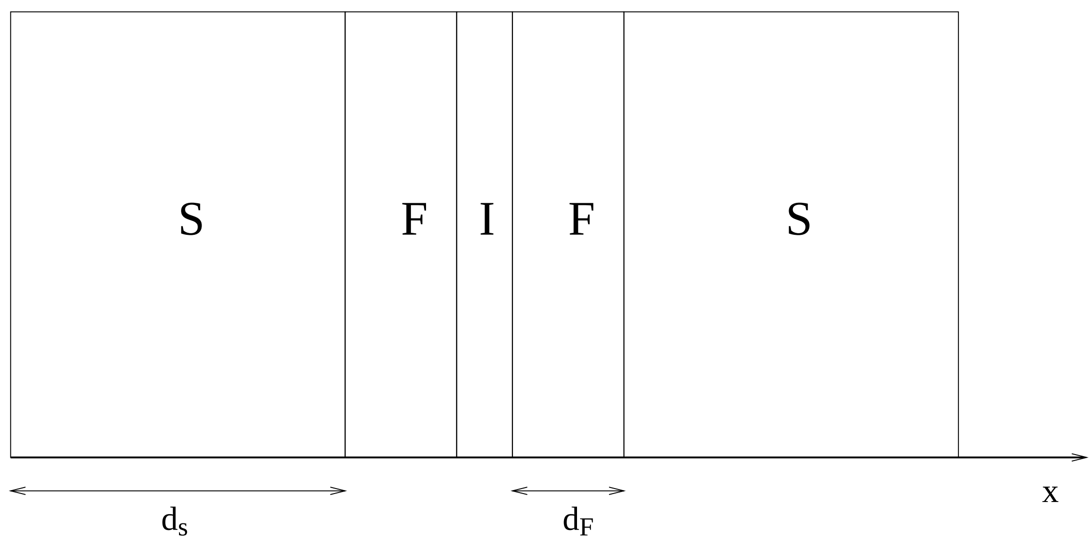
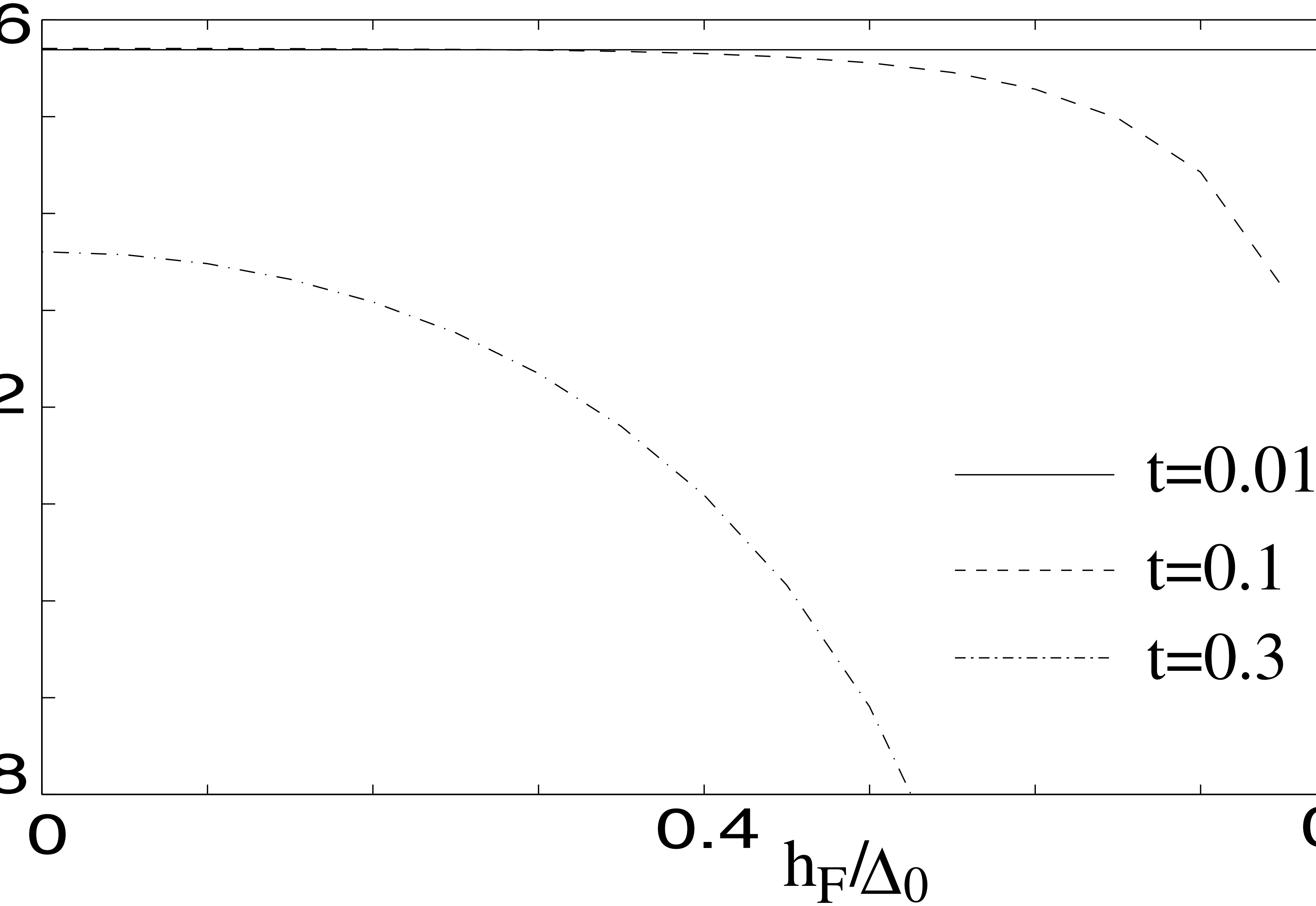


Figure 1:



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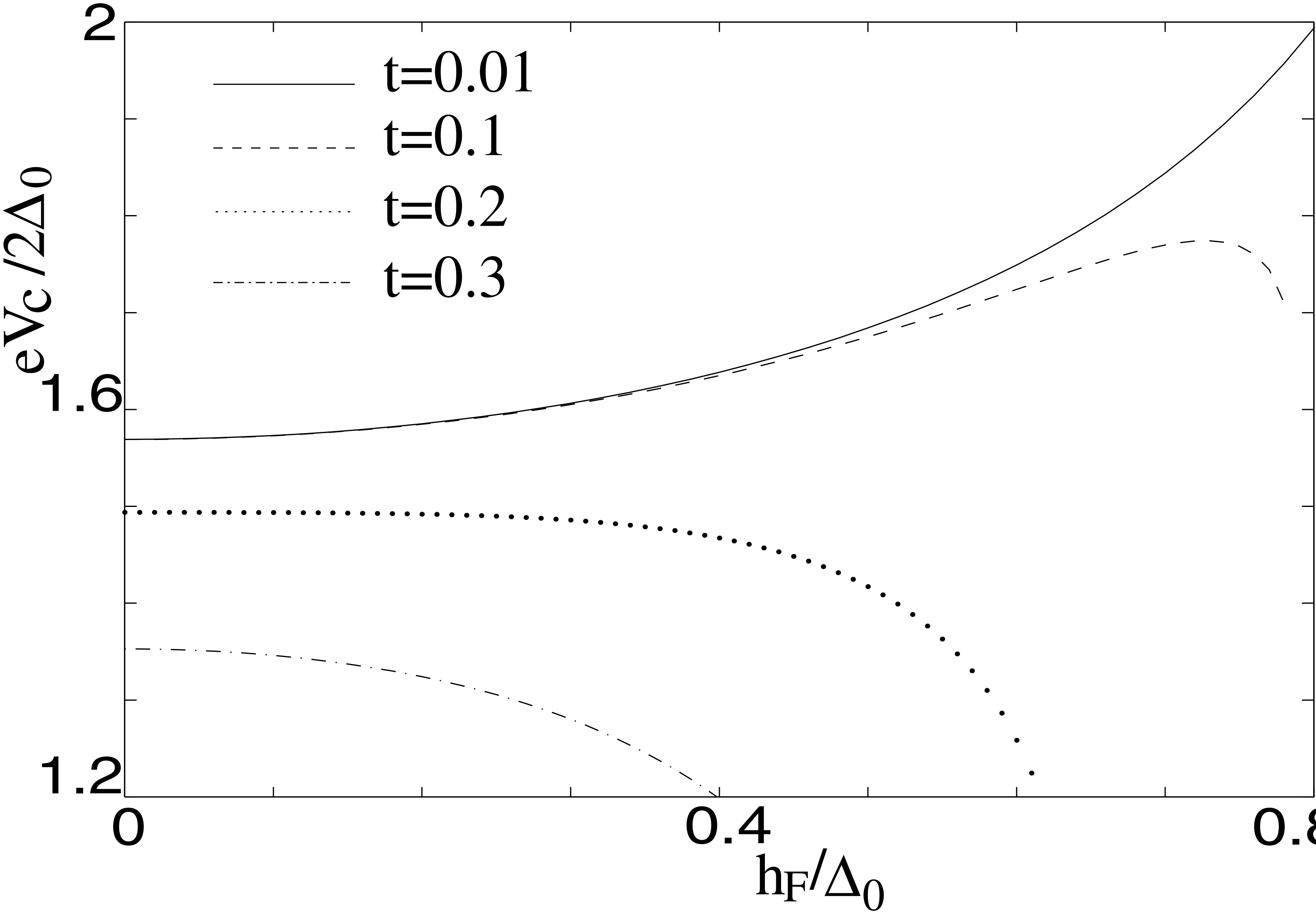

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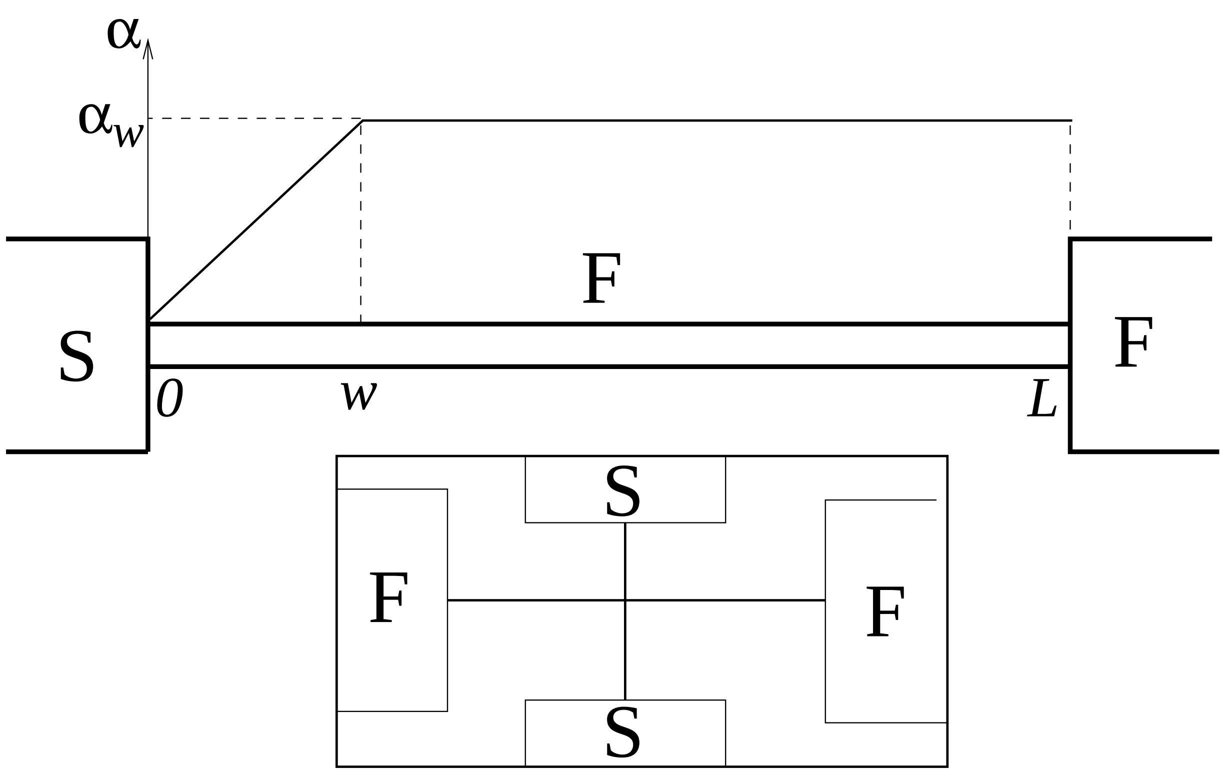

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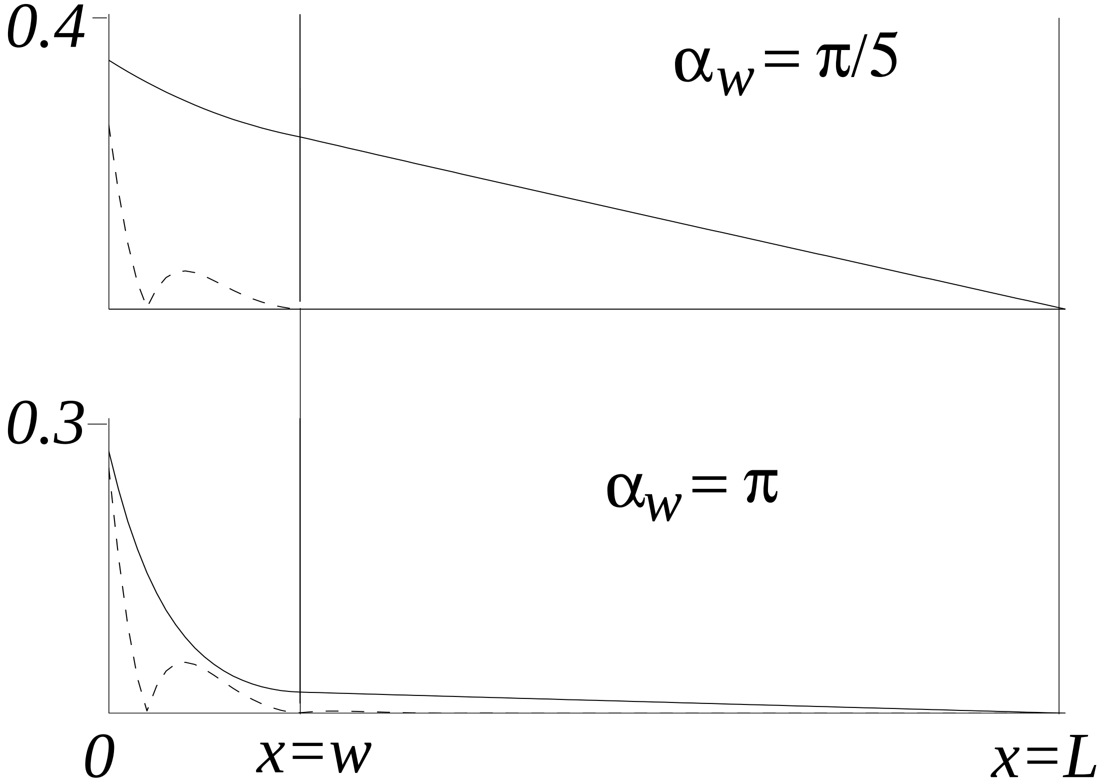

Figure 5:

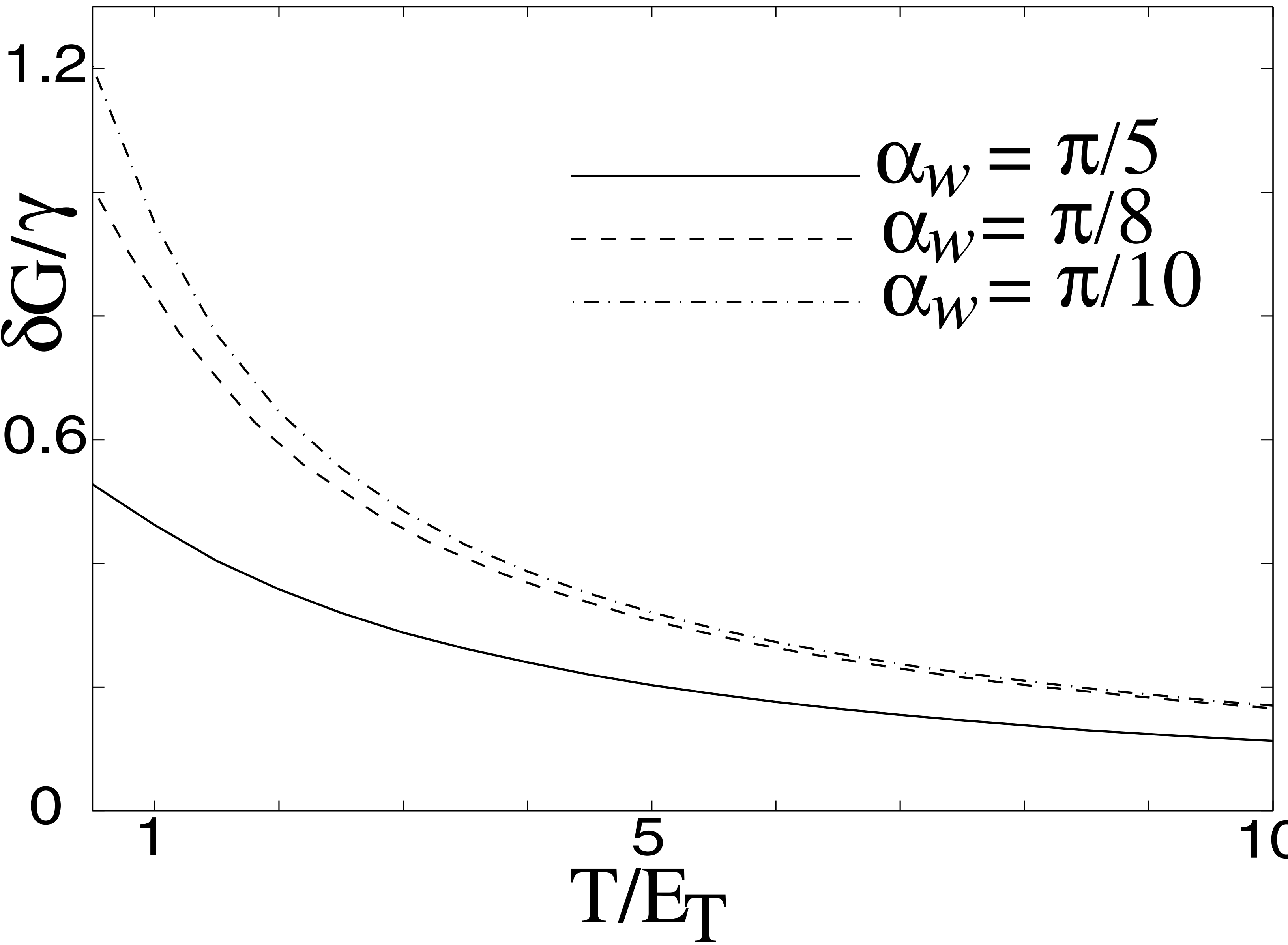

Figure 6: